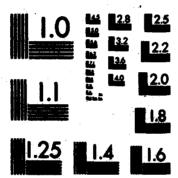


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Title: RESEARCH IN SHELL THEORY

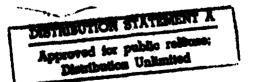
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Dei rection bounds,	
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20. ABSTRACT (Continue on reverse olde II necessary and identify by block number) Summary of research results on formulation of theories of elastic shells,	
plates and beams. Results for unconventional beam buckling problems.	
Results for stress concentrations in shells due to holes or rigid inserts.	
Deflection bounds for unconventional beam problems.	
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DESCRIPTION OF EFFORT Contract NO0014-75-C-0158

The theme of this contractual effort has been the establishment of research results concerning stresses and deformations in "thin" elastic bodies.

The prototype of a thin body is that of a "shell", such as a spherical shell (sometimes designated as dome), or cylindrical shell (straight tube), or toroidal shell (curved tube). The special case of a shell without curvature in its natural state is usually designated as "plate", still other thin bodies are "beams", for which instead of the adjective "thin" one sometimes uses the adjective "slender."

One of the fundamental problems of shell theory is the formulation of two-dimensional theories (or, for beams, of one-dimensional theories) by "direct methods," or as a consequence of a given three-dimensional, linear or non-linear statement of the laws of continuum mechanics. The nature of the step from three dimensions to two or one dimensions may be described briefly by saying that it may depend either on formal asymptotic expansion procedures or on variational procedures having the character of a generalized least-squares approach.

Examples of the direct approach to the solutions oproblem are to be found in items 27, 29 and 34 in the appended list of Technical Reports. Examples of the use of asymptotic expansion procedures are given by items 7 and 12. Examples of the use of the generalized least-squares procedure are items 4, 8, 9, 18 and 28 in the appended list.



In addition to the problem of formulating two-dimensional shell theories and one-dimensional beam theories, some of the work done under the terms of the contract has been concerned with solutions or solution procedures for specific problem of technical interest which can be stated within the framework of established two and one dimensional theories. Among these the following are mentioned specifically.

Novel asymptotic results for two-dimensional sixth order theories of shear deformable plates were presented in three reports (5, 23, 31).

Contributions to the understanding of the distribution of stress and strain in laminated anisotropic (composite materials) cylindrical shells were made in report number 6.

Results concerning finite deflections of circular ring plates and toroidal tubes may be found in reports number 11 and 29.

Several investigations have been concerned with non-conventional problems of axial and lateral beam buckling, including the effects of follower forces, shear deformability, and non-coincident centroidal and shear center locations (2, 16, 24, 32).

Altogether five reports have been devoted to the problem of stress concentrations in shallow spherical shells in states of uniform membrane shear or transverse twisting, due to the effect of a small circular hole or rigid inclusion. These problems were recognized, serendipitously, to represent explicitly solvable problems of shell theory (with previously known classical solutions for the special case of flat plates). They were furthermore found to be particularly suitable examples to illustrate the use of an asymptotic expansion procedure for unsymmetrical shell problems which the Principal Investigator had obtained in 1956. Among

the significant consequences of the analysis of these stress concentration problems was the discovery of the existence of shell problems for which the asymptotic "interior" solution contribution displays physically distinct "near-field" and "far-field" behavior, shifting from "membrane" properties to "inextensional bending" properties, in a manner which depends on the nature of the interaction between "interior" and "edge zone" solution contributions (19, 20, 21, 30).

Contributions to an understanding of the concept of a warping boundary layer in one-dimensional beam theory and the relation of this concept to the problem of determining shear center locations in prismatical beams were made in reports number 15, 17 and 22.

Finally, several reports have dealt with the possibility of determining "upper and lower bounds" for influence coefficients in the theory of orthotropic or anisotropic laminated beams, either based on the theory of plane stress or on the theory of torsion of shafts of varying circular cross section (1, 3, 10, 13, 14).

TECHNICAL REPORTS

Contract N0014-75-C-0158

- 1. Improved Upper and Lower Bounds for Deflections of Orthotropic Cantilever Beams
 Int. J. Solids Structures 11, pp. 961-971, 1975
- 2. Note on a Problem of Beam Buckling
 J. Appl. Math. and Phys. (ZAMP) 26, pp. 839-841, 1975

- 3. On the Determination of Stresses and Deflections for Anisotropic Homogeneous Cantilever Beams
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- 4. Transverse Bending of Laminated Anisotropic Plates
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- 5. On the Theory of Transverse Bending of Elastic Plates Int. J. Solids Structures 12, pp. 545-554, 1976
- 6. On Stretching, Bending, Twisting and Flexure of Cylindrical Shells Int. J. Solids Structures 12, pp. 853-866, 1976
- 7. On Asymptotic Expansions and Error Bounds in the Derivation of Two-Dimensional Shell Theory Studies in Appl. Math. 56, pp. 189-217, 1977
- 8. A Note on Generating Generalized Two-Dimensional Plate and Shell Theories
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- 9. On Small Bending and Stretching of Sandwich-Type Shells Int. J. Solids Structures 13, pp. 1293-1300, 1977
- 10. On Bounds for the Torsional Stiffness of Shafts of Varying Circular Cross Section
 J. Elasticity 8, pp. 221-225, 1978
- 11. A Note on Finite Deflections of Circular Ring Plates
 J. Appl. Math. & Phys. (ZAMP) 29, pp. 698-703, 1978
- 12. Two and Three-Dimensional Results for Rotationally Symmetric Deformations of Circular Cylindrical Shells
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- 13. Effect of Stress-Free Edges in Plane Shear of a Rectangular Orthotropic Region
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- 15. Some Considerations on the Problem of Torsion and Flexure of Prismatical Beams
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- 17. Note on a Nontrivial Simple Example of Higher-Order One-Dimensional Beam Theory
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 Computer Meth. Appl. Mech. & Eng. 20, pp. 203-209, 1979
- 19. On the Transverse Twisting of Shallow Spherical Ring Caps J. Appl. Mech. 47, pp. 101-105, 1980
- 20. On the Effect of a Small Circular Hole on States of Uniform Membrane Shear in Spherical Shells
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- 21. On the Influence of a Rigid Circular Inclusion on the Twisting and Shearing of a Shallow Spherical Shell J. Appl. Mech. 47, pp. 586-588, 1980
- 22. On Torsion and Transverse Flexure of Orthotropic Elastic Plates J. Appl. Mech. 47, pp. 855-860, 1980
- 23. On the Analysis of First and Second-Order Shear Deformation Effects for Isotropic Elastic Plates
 - J. Appl. Mech. 47, pp. 959-961, 1980
- On the Effect of Shear Center Location on the Values of Axial and Lateral Cantilever Buckling Loads for Singly Symmetric Cross-section Beams
 J. Appl. Math. & Phys. (ZAMP) 32, pp. 182-188, 1981
- 25. On Finite Pure Bending of Curved Tubes
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- On a One-Dimensional Theory of Finite Torsion and Flexure of Anisotropic Elastic Plates
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- 27. On Finite Deformations of Space-Curved Beams
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- A Note on Bending of Plates Including the Effects of Transverse Shearing and Normal Strains
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- 29. On the Derivation of Two-Dimensional Strain Displacement Relations For Small Finite Deformations of Shear-Deformable Plates J. Appl. Mech. 49, pp. 232-234, 1982
- 30. Effects of a Rigid Circular Inclusion on States of Twisting and Shearing in Shallow Spherical Shells
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- 31. A Note on the Linear Theory of Shallow Sheardeformable Shells J. Appl. Math: & Phys. (ZAMP) 33, pp. 425-427, 1982
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- 33. Some Remarks on the Problem of Column Buckling Ingenieur-Archiv 52, pp. 115-119, 1982
- 34. A Note on Two-Dimensional Finite-Deformation Theories of Shells Int. J. Non-Linear Mechanics 17, pp. , 1982